First Year of Two Year M.A./M.Sc. Statistics Programme

Semester I

Discipline Specific Core (DSC) Courses

Discipline Specific Core (DSC) Course 1a: Probability Theory

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Prerequisite of the course (if any)
		Lecture (45 Hours)	Tutorial (15 Hours)	Practical (00 Hours)		
DSC 1a: Probability Theory	4	3	1	0	NIL	NIL

Course Objectives:

- To introduce and emphasize the role of measure theory in probability theory.
- To develop the understanding of Weak Law of Large Numbers, Strong Law of Large Numbers and the Central Limit Theorem with their applications.

Course Learning Outcomes: After successfully completing this course, students will be able to apply:

- Concepts of random variables, sigma-fields, probability distributions, and the independence of random variables related to measurable functions.
- Skills in working with measurable functions and sequences of random variables.
- The weak laws of large numbers in practical scenarios.
- The strong laws of large numbers to solve real-world problems.
- The central limit theorem in data analysis and interpretation.
- Principles of convergence and modes of convergence to assess statistical data.
- Characteristic functions, as well as the uniqueness, inversion, and Levy continuity theorems, to advanced probability problems.

Unit I (10 Hours)

Classes of sets, fields, σ -fields, minimal σ -field, Borel σ – field in R^K, sequence of sets, limsup and liminf of a sequence of sets. Measure, Probability measure, properties of a measure, Caratheodory extension theorem (statement only), Lebesgue measures on R^K.

Unit II (11 Hours)

Measurable functions, Random variables, sequence of random variables, Integration of a measurable function with respect to a measure. Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem. Characteristic functions, uniqueness/inversion/Levy continuity theorems.

Unit III (12 Hours)

Markov's, Chebychev's and Kolmogorov's inequalities, Modes of stochastic convergence, Jenson, Liapounov, holder's and Minkowsky's inequalities, Sequence of random variables and modes of convergence (convergence in distribution, in probability, almost surely, and quadratic mean) and their interrelations. Statement of Slutsky's theorem, Borel-Cantelli lemma and Borel 0-1 law.

Unit IV (12 Hours)

Concept of Independence, Laws of large numbers, Chebyshev's and Khinchine's WLLN, necessary and sufficient condition for the WLLN, strong law of large numbers and Kolmogorov's theorem, Central limit theorem, Lindeberg and Levy and Liapunov forms of CLT.

Tutorial:

Tutorial sessions will include at least one activity such as group discussion/presentation/ problem solving exercise based on the material covered in the lectures along with scholastic work related to the conceptual understanding of the subject.

Essential Readings:

1. Ash, R.B. and Doléans-Dade, C.A. (1999). *Probability and Measure Theory*, Academic Press.
2. Bhat, B.R. (1999). *Modern Probability Theory*, New Age International Publishers.
3. Billingsley, P. (2017). *Probability and measure*, John Wiley & Sons.
4. Rohatgi, V.K., and Saleh, A.K.Md.E. (2015). *An introduction to probability and statistics*, John Wiley & Sons.

Suggested Readings:

1. Capinski, M. and Zastawniak, T. (2001). *Probability through problems*, Springer.
2. Chung, K.L. (1974). *A Course in Probability Theory*, Academic Press.
3. Feller, W. (1968). *An Introduction to Probability Theory and its Applications*, Vol. 1, John Wiley & Sons.
4. Parzen, E. (1960). *Modern Probability Theory and its Application*, John Wiley & Sons.