

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?
 (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exist?
 (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N} \sum_{n=1}^N \ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$
- Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2.$

[1] Computer Project 3.8 pp. 187

9. Show that $f(x) = r x (1 - x)$ for $r > 4$ and $f(x) = 6.75 x^2 (1 - x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Project 3.8 pp. 188**
10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions. **[1] Computer Project 4.2 pp. 207**
- a. $x_{n+1} = x_n - y_n + 30$
 $y_{n+1} = x_n + y_n - 20.$
- b. $x_{n+1} = x_n + y_n$
 $y_{n+1} = x_n - y_n.$

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Introduction to Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	Discipline A-3: Differential Equations
---------------------------------------	---	---	---	---	---------------------------------	--

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF DSE-2(ii)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling: Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
 - a. Exponential decay and growth model.
 - b. Lake pollution model (with constant/seasonal flow and pollution concentration).
 - c. Case of single cold pill and a course of cold pills.
 - d. Limited growth of population (with and without harvesting).
 - e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
 - f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
 - g. Ecosystem model of competing species
 - h. Battle model
2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
 - a. $y = a x^2 + b x + c$
 - b. $y = a x^n$
4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
 - a. $y = b x^n$
 - b. $y = b e^{a x}$
 - c. $y = a \ln x + b$
 - d. $y = a x^2$
 - e. $y = a x^3$.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/		