and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

UNIT-II: Continuous Probability Distributions (15 hours)

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

UNIT-III: Central Limit Theorem and Regression Analysis (15 hours)

Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours): Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson, and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

DSE Courses of B.A. (Prog.) Semester-VI
Category-II

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Discrete Dynamical Systems	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**NIL**

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF DSE-2(i)

UNIT-I: Discrete-time Models (12 hours)

Dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, non-homogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, m-cycles, and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
   (a) How long will it take before Rs. 10,000/- is in account?
   (b) During this time how much is deposited and how much comes from interest?

(c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.

[1] Computer Project 2.5 pp. 68

2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.
   (b) What is the unpaid balance on this loan after 4 years.
   (c) Create a time series graph for the unpaid balances each quarter for the loan process.
   [1] Computer Project 2.5 pp. 68
3. Four distinct types of dynamics for any autonomous linear equation:
   $x_{n+1} = a\, x_n + b$ for different values of a and b.
   [1] Dynamics of autonomous linear equation, pp. 74
4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics
   a. $I_{n+1} = I_n - r\, I_n + s\, I_n\,(1 - I_n\, 10^{-6})$
      for: (i) $r = 0.5, s = 0.25$, (ii) $r = 0.5, s = 1.75$, (iii) $r = 0.5, s = 2.0$.
   b. $P_{n+1} = \frac{1}{P_n} + 0.75\, P_n + c$
      for: $(i)\ c = 0$; $(ii)\ c = -1$; $(iii)\ c = -1.25$; $(iv)\ c = -1.38$.
   c. $x_{n+1} = a\, x_n\,(1 - x_n^2)$
      for: $(i)\ a = 0.5$; $(ii)\ a = 1.5$; $(iii)\ a = 2.25$; $(iv)\ a = 2.3$.
   [1] Computer Project 3.2 pp. 110
5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.
   a. $P_{n+1} = r\, P_n\left(1 - \frac{P_n}{5000}\right)$, for: $(i)\ r = 3.4$; $(ii)\ r = 3.5$;
      $(iii)\ r = 3.566$; $(iv)\ r = 3.569$; $(v)\ r = 3.845$.
   b. $P_{n+1} = r\, P_n\, e^{-P_n/1000}$
      for: $(i)\ r = 5$; $(ii)\ r = 10$; $(iii)\ r = 14$; $(iv)\ r = 14.5$; $(v)\ r = 14.75$.
   [1] Computer Project 3.5 pp. 154
6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following
   a. $f_r(x) = r\, x\, e^{-x}$
   b. $f_r(x) = r\, x^2\,(1 - x)$
   c. $f_a(x) = x\,(a - x^2)$
   d. $f_c(x) = \frac{2}{x} + 0.75\, x - c$
   [1] Computer Project 3.6 pp. 164
7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade: (**[1] Computer Project 3.7 pp.175**)
   a. $f_r(x) = r\, x\, e^{-x}$
   b. $f_r(x) = r\, x^2\,(1 - x)$
   c. $f_r(x) = r\, x\, e^{-x^2}$
   d. $f_r(x) = \frac{r\, x}{(x^2+1)^2}$
   e. $f_a(x) = x\,(a - x^2)$

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?
   (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit?
   (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N} \ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$
   Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2$.
   [1] Computer Project 3.8 pp. 187
9. Show that $f(x) = r\,x\,(1-x)$ for $r > 4$ and $f(x) = 6.75\,x^2\,(1-x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Project 3.8 pp. 188**
10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions. **[1] Computer Project 4.2 pp. 207**
    a. $x_{n+1} = x_n - y_n + 30$
       $y_{n+1} = x_n + y_n - 20.$
    b. $x_{n+1} = x_n + y_n$
       $y_{n+1} = x_n - y_n.$

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Introduction to Mathematical Modeling	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**Discipline A-3: Differential Equations**

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF DSE-2(ii)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model

2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$

4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
   a. $y = b\,x^n$
   b. $y = b\,e^{a\,x}$
   c. $y = a\ln x + b$
   d. $y = a\,x^2$
   e. $y = a\,x^3$.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/		

				Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 2(iii)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.