DISCIPLINE SPECIFIC ELECTIVE COURSE – DSE 3: ADVANCED MATHEMATICAL PHYSICS I

Course Title & Code	Credits	Credit distribution of the course			Pre-requisite of the course
		Lecture	Tutorial	Practical	
Advanced Mathematical Physics I DSE – 3	**4**	**4**	**0**	**0**	DSC courses of Mathematical Physics I and Mathematical Physics III

LEARNING OBJECTIVES

The objective of the course is to impart the concept of calculus of variation and generalized mathematical constructs in terms of algebraic structures mainly vector spaces. Both concepts are extremely useful in physics, engineering, machine learning, economics and even life sciences and social sciences. While linear algebra studies linear vector spaces, linear transformations, and the matrices, calculus of variation is an important mathematical tool in optimization. This course is intended to provide a solid foundation in both topics as used by physicists and has direct applications in classical and quantum mechanics.

LEARNING OUTCOMES

After completing this course, student will be able to,

- Understand algebraic structures in n-dimension and basic properties of the linear vector spaces.
- Represent linear transformations as matrices and understand basic properties of matrices.
- Determine the eigenvalues and eigenvectors of matrices and diagonalize the matrices.
- Determine orthogonal basis for a vector space using Gram-Schmidt procedure.
- Understand the concept of dual spaces and inner product spaces.
- Apply vector spaces and matrices in the quantum world.
- Understand what functionals are and appreciate their applications.
- Solve Euler-Lagrange equations for simple cases.
- Apply the techniques of calculus of variation to real world problems.

SYLLABUS OF DSE - 3

THEORY COMPONENT

Unit – I **(18 Hours)**

Calculus of Variation: Functionals and extrema, Euler's equation for (i) one independent and one dependent variable, (ii) several dependent variables and (iii) several independent variables; variable end-point problems; application to problems (e.g. geodesics, catenary, minimum area of soap film, brachistochrone, Fermat's principle, Laplace equation etc.); generalised coordinates and concept of Lagrangian; Hamilton's principle, Euler-Lagrange's equations of motion and its applications to physics problems (e.g. simple pendulum and one dimensional harmonic oscillator and other problems)

Unit – II **(12 Hours)**

Vector Spaces as Algebraic Structures: Definition and examples of groups, rings, fields and vector spaces; real and complex fields, use of ket notation $|\alpha >$ for vectors

Subspaces, linear combination of vectors, linear dependence and independence of vectors, span of a subset of vectors, bases and dimension of vector space, direct sum of spaces, representation of vectors as column matrix with R_n as example

Inner Product Spaces: Inner product of vectors ($< \alpha|\beta >$) and norm of a vector, Euclidean spaces and unitary spaces, Cauchy-Schwartz inequality, concept of length and distance, metric spaces. Hilbert Space (definition only); linear functional, dual space, dual basis ($< \alpha|$ notation), orthogonality of vectors, orthonormal basis, Gram-Schmidt procedure to construct an orthonormal basis

Unit – III **(18 Hours)**

Linear Transformation: Linear mappings and examples, homomorphism and isomorphism of vector space, rank and nullity of a linear mapping, range space and Kernel (null space) of a linear mapping, non-singular transformations

Matrices as Representations: Matrix representation of linear transformations, composition of linear transformations and matrix multiplication, linear algebra. Algebra of matrices, determinant and trace of matrix and their properties, non-singular matrices, rank of a matrix and invertibility of matrices, direct sum and direct product of matrices. Change of basis transformation, similar matrices; transpose and adjoint of a linear transformation, self-adjoint operators; symmetric and Hermitian matrices; preservation of norms by orthogonal and unitary transformations

Unit – IV **(12 Hours)**

Eigen-values and Eigenvectors: Eigen-values and eigen vectors of a transformation and corresponding matrix representation; Cayley-Hamilton theorem (statement only), its applications like inverse and powers of a matrix; eigen systems of Hermitian and unitary matrices; diagonalization of matrices; normal matrices; simultaneous diagonalizability of two matrices

Use of matrices in solving coupled linear first order ordinary differential equations with constant coefficients, minimal polynomial, functions of a matrix.

References:

Essential Readings:

1) Mathematical Methods for Physicists, G. Arfken, H. Weber and F. E. Harris, 7th edition, 2012, Elsevier
2) Applied Mathematics for Engineers and Physicists, L. A. Pipes and L. R. Harvill, 1970, McGraw-Hill Inc
3) Introduction to Matrices and Linear Transformations, D. T. Finkbeiner, 2011, Dover Publications
4) Schaum's Outline of Theory and Problems of Linear Algebra, S. Lipschutz and M. Lipson, 2017, McGraw Hill Education
5) Linear Algebra, S. H. Friedberg, A. J. Insel, and L. E. Spence, 2022, Pearson Education
6) Calculus of Variations, I. M. Gelfand and S. V. Fomin, 2000, Dover Publications

Additional Readings:

1) Elementary Linear Algebra with Supplemental Applications, H. Anton and C. Rorres,

2016, Wiley Student Edition
2) A Physicist's Introduction to Algebraic Structures: Vector Spaces, Groups, Topological Spaces and More, P. B. Pal, 2019, Cambridge University Press
3) Matrices and Tensors in Physics: A.W. Joshi, 2017, New Age International Pvt. Ltd.
4) An Introduction to Linear Algebra and Tensors, M. A. Akivis, V. V. Goldberg, Richard and Silverman, 2012, Dover Publications
5) Linear Algebra and Applications, D. C. Lay, 2002, Pearson Education India
6) Vector Spaces and Matrices in Physics, M. C. Jain, 2000, Narosa
7) Mathematical Methods for Physics and Engineering, K. F. Riley and M. P. Hobson, 2018, Cambridge University Press