Congruence relation and its basic properties, Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables; Fermat's little theorem and its generalization, Wilson's theorem and its converse; Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions, The greatest integer function; Euler's Phi-function and its properties.

Unit – 3 **(12 hours)**

Public Key Encryption and Numbers of Special Form

Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique; Introduction to perfect numbers, Mersenne numbers and Fermat numbers.

Essential Reading

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

Suggestive Readings

- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE - DSE-1(iii): THEORY OF EQUATIONS AND SYMMETRIES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Theory of Equations and Symmetries	4	3	1	0	Class X pass with Mathematics	Nil

Learning Objectives

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

Learning Outcomes

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

SYLLABUS OF DSE-1(iii)

Unit – 1 (18 hours)

Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions.

Unit – 2 (12 hours)

Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit – 3 (15 hours)

Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as $\sum\alpha^2\beta,\ \sum\alpha^2\beta^2, \sum\alpha^2\beta\gamma, \sum\frac{1}{\alpha^2\beta\gamma}, \sum\alpha^{-3}, \sum(\beta+\gamma-\alpha)^2,\ \sum\frac{\alpha^2+\beta\gamma}{\beta+\gamma},\ldots$ of polynomial equations; Transformation of equations by symmetric functions and in general.

Essential Readings

1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

Suggestive Readings

- Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

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