Syllabi of Semester - VIII based on UGCF - 2022

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VIII

DISCIPLINE SPECIFIC CORE COURSE (DSC)– 20: FIELD THEORY AND GALOIS EXTENSION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Field Theory and Galois Extension	4	3	1	0	Class XII pass with Mathematics	Group Theory, Ring Theory

Learning Objectives: The objective of the course is to introduce:

- Tools of field theory such as field extensions, splitting fields, normal extensions, separability, and separable extensions.
- Galois extensions and the Fundamental theorem of Galois theory.
- Link between group theory and the roots of polynomials, developed by Galois, to solve the problem of solvability of polynomial equations by radicals.
- Some applications, such as cyclotomic polynomials, finite fields, and simple extensions.

Learning Outcomes: This course will enable the students to:

- Identify and construct examples of fields, distinguish between algebraic and transcendental extensions, and characterize normal extensions in terms of splitting fields.
- Identify and characterize separable extensions, define Galois extensions, construct examples of automorphism groups of a field as well as prove the fundamental theorem of Galois theory.
- Use the Galois theory of equations to prove that a polynomial equation over a field is solvable by radicals if and only if its Galois group is solvable and hence deduce that a general quintic equation is not solvable by radicals.
- Define cyclotomic polynomials and find its Galois group using roots of unity, classify finite fields and prove that every finite separable extension is simple.

SYLLABUS OF DSC-20

UNIT – I: Field Extensions **(15 hours)**

Fields and prime subfields, Field extensions, Degree of field extensions, Tower theorem, Algebraic and transcendental elements, Algebraic and transcendental extensions, Monomorphism of field extensions, Ruler and compass constructions, Splitting fields, Extensions of monomorphisms, Uniqueness of splitting field.

UNIT – II: Galois Extensions and the Fundamental Theorem (15 hours)

Normal extensions, Separability and separable extensions, Monomorphisms and automorphisms of field extension, Galois extensions, Automorphism/Galois groups and fixed fields, Galois theory of polynomials, The fundamental theorem of Galois theory.

UNIT – III: Some Applications and Solvability by Radicals (15 hours)

The Discriminant, Cyclotomic polynomials, extensions and its Galois group, Solution by radicals, Existence and Uniqueness of finite fields, Simple extensions, and the primitive element theorem.

Essential Readings

1. Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.
2. Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.). Wiley.

Suggestive Readings

- Stewart, Ian (2022). Galois Theory (5th ed.). CRC Press. Chapman and Hall.
- Cox, David A. (2012). Galois Theory (2nd ed.). John Wiley & Sons.
- Cohn, P. M. (2003). Basic Algebra, Springer International Edition.