DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(v): INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Equations and Calculus of Variations	4	3	1	0	Class XII pass with Mathematics	Ordinary, and Partial Differential Equations, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Familiarize the learner with methods for solving Volterra and Fredholm integral equations.
- Know the determination of extremum of functional, necessary condition for an extremum, Euler's equation, and its generalization.

Learning Outcomes: This course will enable the students to:

- Compute the solutions to Volterra integral equations by method of resolvent kernel, method of successive approximations, method of Laplace transform, system of Volterra integral equations and integro-differential equation.
- Determine the solutions of Fredholm integral equations and derivation of Hilbert-Schmidt theorem.
- Understand the formulation of variational problems, the variation of a functional and its properties, extremum of functional, necessary condition for an extremum.

SYLLABUS OF DSE-6(v)

UNIT – I: Volterra Integral Equations **(12 hours)**

Integral equations, Introduction and relation with linear differential equations; Volterra integral equations and its solutions, Method of resolvent kernel, Method of successive approximations, Convolution type of equation, Method of Laplace transform, System of Volterra integral equations, Integro-differential equation, Abel's integral equation and its generalizations.

UNIT – II: Fredholm Integral Equations **(18 hours)**

Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels, Eigenvalues and eigen functions and their properties, Hilbert-Schmidt theorem, Nonhomogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

UNIT – III: Calculus of Variations **(15 hours)**

Variational problems, Variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a functional and variable end point problem, Sufficient conditions for the extremum of a functional.

Essential Readings

1. Gelfand, I. M. and Fomin, S.V. (2000). Calculus of Variations. Dover Publications, Inc.
2. Krasnov, M., Kiselev, A. and Makarenko, G. (1971). Problems and Exercises Integral Equations, Mir Publication Moscow.
3. Logan, J. David (1987). Applied Mathematics: A Contemporary Approach, John Wiley & Sons, Inc.

Suggestive Readings

- Hildebrand, F. B. (1992). Methods of Applied Mathematics (2nd ed.). Dover Publications.
- Zemyan, Stephen M. (2012). The Classical Theory of Integral Equations: A Concise Treatment. Birkhäuser.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(vi): MACHINE LEARNING: A MATHEMATICAL APPROACH

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Machine Learning: A Mathematical Approach	4	3	0	1	Class XII pass with Mathematics	Basic Knowledge of Python

Learning Objectives: The main objective of this course is to:

- Gain mathematical insights into the functioning of popular methods of Regression, Classification, Clustering and Dimension reduction.
- Understand the mathematical framework of learning and apply it to assess the performance of a number of regression, classification and density estimation algorithms
- Detect overfitting and employ regularization techniques to control it.

Learning Outcomes: This course will enable the students to:

- Learn how to build popular models of regression and classification including Linear regression, Polynomial regression, Logistic classifier, Support vector machine, Decision Tree, Random forests, Naïve Bayes classifier.
- Evaluate the performance of models on test data through analytical techniques (VC bounds and dimension) and Cross-validation to facilitate model selection and feature selection.
- Improve model performance by controlling overfitting through regularization techniques like Ridge and Lasso.