

GE-6(i)

GENERIC ELECTIVES (GE-6(i)): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/Practice		
Introduction to Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	GE-3(i): Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF GE-6(i)

UNIT-I: Compartmental Models **(15 hours)**

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis **(15 hours)**

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation **(15 hours)**

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling: Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
 - a. Exponential decay and growth model.
 - b. Lake pollution model (with constant/seasonal flow and pollution concentration).
 - c. Case of single cold pill and a course of cold pills.
 - d. Limited growth of population (with and without harvesting).
 - e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
 - f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
 - g. Ecosystem model of competing species
 - h. Battle model
2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
 - a. $y = a x^2 + b x + c$
 - b. $y = a x^n$
4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
 - a. $y = b x^n$
 - b. $y = b e^{ax}$
 - c. $y = a \ln x + b$
 - d. $y = a x^2$
 - e. $y = a x^3$.