COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VIII COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-8(i)): RINGS AND FIELDS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Rings and Fields	4	3	1	0	Class XII pass with Mathematics	Abstract Algebra

Learning Objectives: The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

Learning Outcomes: This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

SYLLABUS OF GE-8(i)

UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

UNIT-II: Polynomial Rings and Field Extensions (15 hours)

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in *n* variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

UNIT-III: Algebraic Extensions **(15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions; Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field; Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

Suggestive Readings

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

GENERIC ELECTIVES (GE-8(ii)): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type $Rr + Ss + Tt = V$.
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.