

**DISCIPLINE SPECIFIC ELECTIVE COURSE - DSE-1(iii):  
THEORY OF EQUATIONS AND SYMMETRIES**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Theory of Equations and Symmetries	4	3	1	0	Class X pass with Mathematics	Nil

**Learning Objectives**

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

## Learning Outcomes

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

## SYLLABUS OF DSE-1(iii)

### Unit – 1 (18 hours)

#### Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the  $n$ th roots of unity and symmetries of the solutions.

### Unit – 2 (12 hours)

#### Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

### Unit – 3 (15 hours)

#### Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric

functions such as  $\sum \alpha^2 \beta$ ,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ , ... of polynomial equations; Transformation of equations by symmetric functions and in general.

#### Essential Readings

1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>

#### Suggestive Readings

- Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.