

## DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-III

### DISCIPLINE SPECIFIC ELECTIVE -1(i): COMBINATORICS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Combinatorics	4	3	1	0	Class XII pass with Mathematics	Nil

#### Learning Objectives

The primary objective of this course is to:

- Introduce various techniques of permutations, combinations and inclusion-exclusion.
- Learn basic models of generating functions and recurrence relations in their application to the theory of integer partitions.

#### Learning Outcomes

After completing the course, student will:

- Enhance the mathematical logical skills by learning different enumeration techniques.
- Be able to apply these techniques in solving problems in other areas of mathematics.
- Be trained to provide reasoning and arguments to justify conclusions.

### SYLLABUS OF DSE-1(i)

#### Unit - 1

(15 hours)

##### Basics of Combinatorics

Basic counting principles, Permutations and Combinations (with and without repetitions), Binomial coefficients, Multinomial coefficients, Counting subsets of size  $k$ ; Set-partitions, The inclusion-exclusion principle and applications.

#### Unit - 2

(18 hours)

##### Generating Functions and Recurrence Relations

Generating functions: Generating function models, Calculating coefficients of generating functions, Polynomial expansions, Binomial identity, Exponential generating functions.

Recurrence relations: Recurrence relation models, Divide-and-conquer relations, Solution of linear recurrence relations, Solutions by generating functions.

#### Unit – 3

(12 hours)

##### Partition

Partition theory of integers: Ordered partition, Unordered partition, Ferrers diagram, Conjugate of partition, Self-conjugate partition, Durfee square, Euler's pentagonal theorem.

#### Essential Readings

1. Sane, Sharad S. (2013). Combinatorial Techniques. Hindustan Book Agency (India).
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.

### Suggested Readings

- Brualdi, Richard A. (2009). Introductory Combinatorics (5th ed.). Pearson Education Inc.
- Cameron, Peter J. (1994). Combinatorics: Topics, Techniques, Algorithms. Cambridge University Press.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-1(ii): ELEMENTS OF NUMBER THEORY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Number Theory	4	3	1	0	Class XII pass with Mathematics	Nil

### Learning Objectives

The primary objective of this course is to introduce:

- The Euclidean algorithm and linear Diophantine equations, the Fundamental theorem of arithmetic and some of the open problems of number theory viz. the Goldbach conjecture.
- The modular arithmetic, linear congruence equations, system of linear congruence equations, arithmetic functions and multiplicative functions, e.g., Euler's Phi-function.
- Introduction of the simple encryption and decryption techniques, and the numbers of specific forms viz. Mersenne numbers, Fermat numbers etc.

### Learning Outcomes

This course will enable the students to:

- Get familiar with the basic number-theoretic techniques.
- Comprehend some of the open problems in number theory.
- Learn the properties and use of number-theoretic functions and special types of numbers.
- Acquire knowledge about public-key cryptosystems, particularly RSA.

### SYLLABUS OF DSE-1(ii)

#### Unit – 1

(12 hours)

#### Divisibility and Prime Numbers

Revisiting: The division algorithm, divisibility and the greatest common divisor. Euclid's lemma; The Euclidean algorithm, Linear Diophantine equations; The Fundamental theorem of Arithmetic, The sieve of Eratosthenes, Euclid theorem and the Goldbach conjecture; The Fibonacci sequence and its nature.

#### Unit – 2

(21 hours)

#### Theory of Congruences and Number-Theoretic Functions

Congruence relation and its basic properties, Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables; Fermat's little theorem and its generalization, Wilson's theorem and its converse; Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions, The greatest integer function; Euler's Phi-function and its properties.

**Unit – 3**

**(12 hours)**

**Public Key Encryption and Numbers of Special Form**

Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique; Introduction to perfect numbers, Mersenne numbers and Fermat numbers.

**Essential Reading**

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

**Suggestive Readings**

- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

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**DISCIPLINE SPECIFIC ELECTIVE COURSE - DSE-1(iii):  
THEORY OF EQUATIONS AND SYMMETRIES**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Theory of Equations and Symmetries	4	3	1	0	Class X pass with Mathematics	Nil

**Learning Objectives**

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

## Learning Outcomes

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

## SYLLABUS OF DSE-1(iii)

### Unit – 1 (18 hours)

#### Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the  $n$ th roots of unity and symmetries of the solutions.

### Unit – 2 (12 hours)

#### Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

### Unit – 3 (15 hours)

#### Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric

functions such as  $\sum \alpha^2 \beta$ ,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ , ... of polynomial equations; Transformation of equations by symmetric functions and in general.

#### Essential Readings

1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>

#### Suggestive Readings

- Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

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