## DISCIPLINE SPECIFIC ELECTIVE COURSE-4(ii): INTRODUCTION TO MATHEMATICAL MODELING

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course title & Code | Credits | Credit distribution of the course | | | Eligibility criteria | Pre-requisite of the course (if any) |
|---|---|---|---|---|---|---|
| | | Lecture | Tutorial | Practical/ Practice | | |
| Introduction to Mathematical Modeling | 4 | 3 | 0 | 1 | Class XII pass with Mathematics | Discipline A-3: Differential Equations |

**Learning Objectives:** The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

**Learning Outcomes:** The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

### SYLLABUS OF DSE-4(ii)

**UNIT-I: Compartmental Models (15 hours)**

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

**UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)**

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

**UNIT-III: Analytic methods of model fitting and Simulation (15 hours)**

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

**Essential Readings**


1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.


**Suggestive Readings**


- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.


**Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:**

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model

2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$

4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
    a. $y = b\,x^n$
    b. $y = b\,e^{a\,x}$
    c. $y = a \ln x + b$
    d. $y = a\,x^2$
    e. $y = a\,x^3$.