

**DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-VII****Category-III****DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(i): ADVANCED LINEAR ALGEBRA****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra

**Learning Objectives:** The objective of the course is to introduce:

- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

**Learning Outcomes:** This course will enable the students to:

- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.
- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

**SYLLABUS OF DSE-5(i)****UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)**

The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces, and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

**UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)**

Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

**UNIT-III: Class of Operators and Their Properties (15 hours)**

Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

**Essential Reading**

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

**Suggestive Readings**

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-5(ii): ELEMENTS OF METRIC SPACES****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Metric Spaces	4	3	1	0	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

**Learning Outcomes:** This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.

**SYLLABUS OF DSE-5(ii)****UNIT-I: Topology of Metric Spaces (18 hours)**

Inequalities, Definition and examples, Sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Closed set, Closure of a set; Subspaces.

**UNIT-II: Continuity and Uniform Continuity in Metric Spaces (15 hours)**

Continuous mappings, Sequential criterion, and other characterizations of continuity; Uniform continuity; Homeomorphism, isometry, and equivalent metrics.

**UNIT-III: Connected and Compact Spaces (12 hours)**

Connected subsets of  $\mathbb{R}$ , Connectedness and continuous mappings; Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

**Essential Reading**

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

**Suggestive Reading**

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-5(iii): MATHEMATICAL DATA SCIENCE****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Data Science	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of R/Python, Probability and Statistics

**Learning Objectives:** The main objective of this course is to:

- Introduce various types of data and their sources, along with steps involved in data science case-study, including problems with data and their rectification and creation methods.
- Cover dimensionality reduction techniques, clustering algorithms and classification methods.

**Learning Outcomes:** The course will enable the students to:

- Gain a comprehensive understanding of data science, its mathematical foundations including practical applications of regression, principal component analysis, singular value decomposition, clustering, support vector machines, and  $k$ -NN classifiers.
- Demonstrate data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation and regularization using R/Python.
- Use real-world datasets to practice dimensionality reduction techniques such as PCA, SVD, and multidimensional scaling using R/Python.

## SYLLABUS OF DSE-5(iii)

### UNIT-I: Principles of Data Science (12 hours)

Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas, anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

### Unit-II: Mathematical Foundations (15 hours)

Model driven data in  $R^n$ , Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling; Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances; Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

### Unit-III: Dimensionality Reduction, Clustering and Classification (18 hours)

Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best  $k$ -rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis; Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez’s algorithm for  $k$ -center clustering, Lloyd’s algorithm for  $k$ -means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering; Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and  $k$ -nearest neighbors ( $k$ -NN) classifiers.

### Essential Readings

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (<https://mathfordata.github.io/>).

### Suggestive Readings

- Frank Emmert-Streib, et al. (2022). Mathematical Foundations of Data Science Using R. (2nd ed.). De Gruyter Oldenbourg.
- Wes McKinney. (2022). Python for Data Analysis (3rd ed.). O'Reilly.
- Wickham, Hadley, et al. (2023). R for Data Science (2nd ed.). O'Reilly.

### Practical (30 hours)- Practical work to be performed in Computer Lab using R/Python:

1. To explore different types data (nominal, ordinal, interval, ratio) and identify their properties.
2. To deal with dirty and missing data, such as imputation, deletion, and data normalization.
3. Use the real-world datasets (<https://data.gov.in/>) to demonstrate the following:
  - e) Data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation, and regularization.
  - f) Dimensionality reduction techniques such as principal component analysis, singular value decomposition (SVD), and multidimensional scaling.
  - g) Clustering algorithms such as  $k$ -means, hierarchical, and density-based clustering and evaluate the quality of the clustering results.
  - h) Classification methods such as linear classifiers, support vector machines (SVM), and  $k$ -nearest neighbors ( $k$ -NN).

## DISCIPLINE SPECIFIC ELECTIVE COURSE-5(iv): INTEGRAL TRANSFORMS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Integral Transforms</b>	4	3	1	0	Class XII pass with Mathematics	Differential Equations, Elementary Mathematical Analysis

### Learning Objectives: Primary objective of this course is to introduce:

- The basic idea of integral transforms of functions and their applications through an introduction to Fourier series expansion of a periodic function.
- Fourier transform and Laplace transform of functions of a real variable with applications to solve ODE's and PDE's.

### Learning Outcomes: The course will enable the students to:

- Understand the Fourier series associated with a periodic function, its convergence, and the Gibbs phenomenon.
- Compute Fourier and Laplace transforms of classes of functions.
- Apply techniques of Fourier and Laplace transforms to solve ordinary and partial differential equations and initial and boundary value problems.

## **SYLLABUS OF DSE-5(iv)**

### **UNIT-I: Fourier Series and Integrals**

**(18 hours)**

Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

### **UNIT-II: Integral Transform Methods**

**(15 hours)**

Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

### **UNIT-III: Applications of Integral Transforms**

**(12 hours)**

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

### **Essential Readings**

1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

### **Suggestive Readings**

- Baidyanath Patra (2018). An Introduction to Integral Transforms. CRC Press.
- Joel L. Schiff (1999). The Laplace Transform-Theory and Applications. Springer.
- Rajendra Bhatia (2003). Fourier Series (2nd ed.). Texts and Readings in Mathematics, Hindustan Book Agency, Delhi.
- Yitzhak Katznelson (2004). An Introduction to Harmonic Analysis (3rd ed.). Cambridge University Press.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-5(v): RESEARCH METHODOLOGY**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

**Learning Outcomes:** The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

### **SYLLABUS OF DSE-5(v)**

#### **UNIT- I: How to Learn, Write, and Research Mathematics (17 hours)**

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

#### **UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)**

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

#### **UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)**

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics;

Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

### Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (<https://publicationethics.org/>)
3. Declaration on Research Assessment.  
[https://en.wikipedia.org/wiki/San\\_Francisco\\_Declaration\\_on\\_Research\\_Assessment](https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment)
4. Evaluating Journals using journal metrics;  
(<https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874>)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (<https://www.d.umn.edu/~jgallian/goodPPTalk.pdf>). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group.  
(<https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf>)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

**Practical (30 hours):** Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.