

**COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VIII COURSES OFFERED  
BY DEPARTMENT OF MATHEMATICS**

**Category-IV**

**GENERIC ELECTIVES (GE-8(i)): RINGS AND FIELDS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Rings and Fields</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Abstract Algebra</b>

**Learning Objectives:** The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

**Learning Outcomes:** This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

**SYLLABUS OF GE-8(i)**

**UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)**

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

**UNIT-II: Polynomial Rings and Field Extensions (15 hours)**

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in  $n$  variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

**UNIT-III: Algebraic Extensions****(15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions; Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field; Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

**Essential Readings**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

**Suggestive Readings**

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

**GENERIC ELECTIVES (GE-8(ii)): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

**Learning Objectives:** The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

**Learning Outcomes:** The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type  $Rr + Ss + Tt = V$ .
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

**SYLLABUS OF GE-8(ii)****UNIT–I: First-order Partial Differential Equations (18 hours)**

Review of basic concepts: Origins of first-order PDEs, Lagrange’s method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit’s method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi’s method for solving nonlinear PDE with three independent variables.

**UNIT – II: Second-order Partial Differential Equations (15 hours)**

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge’s method of integrating nonlinear second-order PDE of type  $Rr + Ss + Tt = V$  with variable coefficients.

**UNIT – III: Applications of Partial Differential Equations (12 hours)**

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d’Alembert’s solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

**Essential Readings**

- 1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
- 3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

**Suggestive Readings**

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

**GENERIC ELECTIVES (GE-8(iii)): ELEMENTS OF COMPLEX ANALYSIS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Complex Analysis	4	3	1	0	Class XII pass with Mathematics	Metric Spaces, Multivariate Calculus

**Learning Objectives:** The primary objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant examples.
- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

**Learning Outcomes:** The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

## SYLLABUS OF GE-8(iii)

### Unit-I: Analytic Functions

(15 hours)

Basic properties of complex numbers and their exponential form; Limits, continuity, and partial derivatives of functions of two variables. Limits, continuity, and partial derivatives of functions of a complex variable; Cauchy-Riemann Equations, Sufficient conditions for differentiability; Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

### Unit-II: Complex Integrals

(15 hours)

Derivatives of functions, Definite integrals of functions, Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals, Antiderivatives; Statement of Cauchy-Goursat theorem; Cauchy integral formula and its extension, Cauchy's inequality, Liouville's theorem and the fundamental theorem of algebra.

### Unit-III: Series and Residues

(15 hours)

Convergence of sequences and series of complex numbers; Taylor, and Laurent series with examples; Isolated singular points, Residues, Cauchy's residue theorem; Types of isolated singular points, Residues at poles and its examples.

### Essential Reading

1. Brown, James Ward & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

### Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.

## GENERIC ELECTIVES (GE-8(iv)): OPTIMIZATION TECHNIQUES

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

**Learning Objectives:** The primary objective of this course is to introduce:

- Nonlinear optimization problems
- Transshipment and dynamic programming problems
- Integer Programming, fractional programming problems
- Convex and generalized convex functions and their properties

**Learning Outcomes:** This course will enable the students to:

- Nonlinear programming problems and their applications
- Method to solve fractional programming problems with linear constraints
- Methods to solve dynamic programming problems using recursive computations

### SYLLABUS OF GE-8(iv)

#### UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

#### UNIT-II: Integer Programming Problems (15 hours)

Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

#### UNIT-III: Nonlinear Programming Problems (15 hours)

Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

#### Essential Readings

1. Chandra, Suresh, Jayadeva, and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

#### Suggestive Reading

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.